Passive Circuit Elements

Capacitors and Inductors

- This lecture is all about energy stored in Electric Fields and Magnetic Fields, and how that can be used in electric circuits when the Current is Changing.

But First: How is your payload planning going?

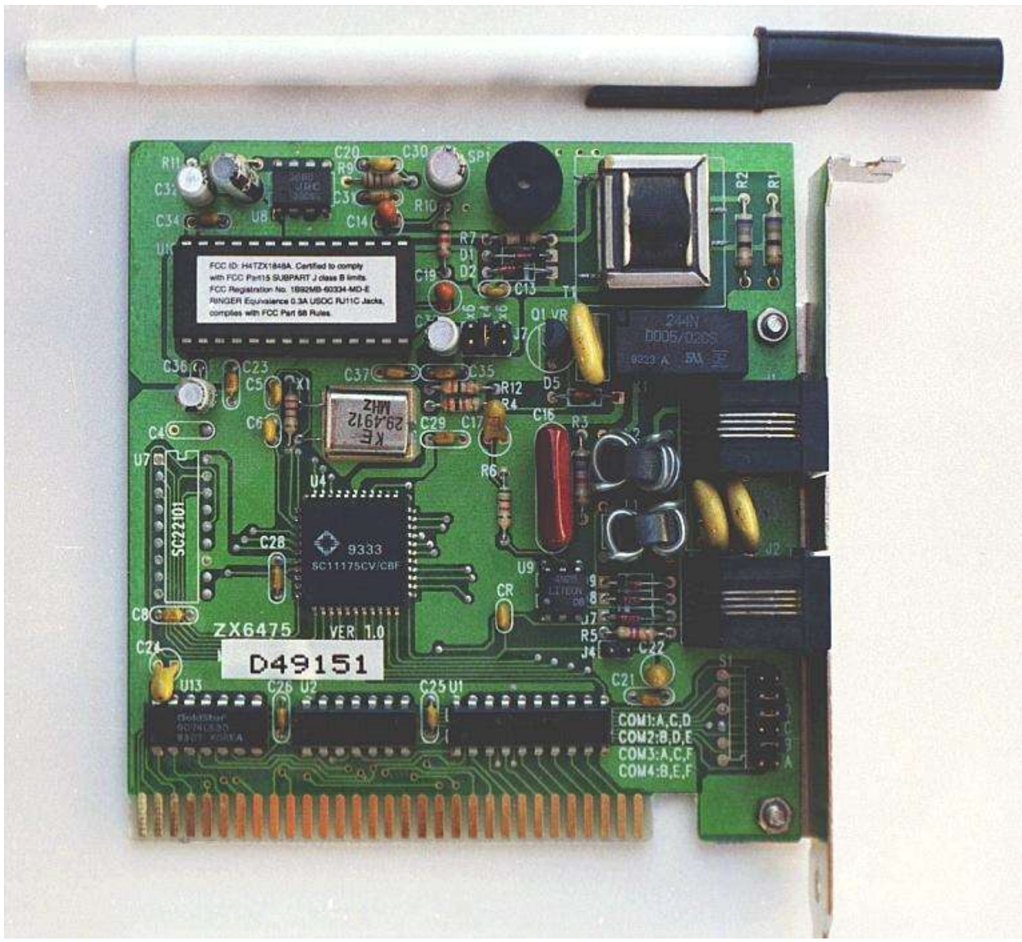
- 1. update your wiki with your team plans
- 2 Project Concept needs to be posted
- 3. Final approval in 1.5 weeks (by April 13)

Let me ask you a question:

Are you spending enough time outside of this class to succeed?

This is a 5 credit class: how much homework do you think is appropriate?

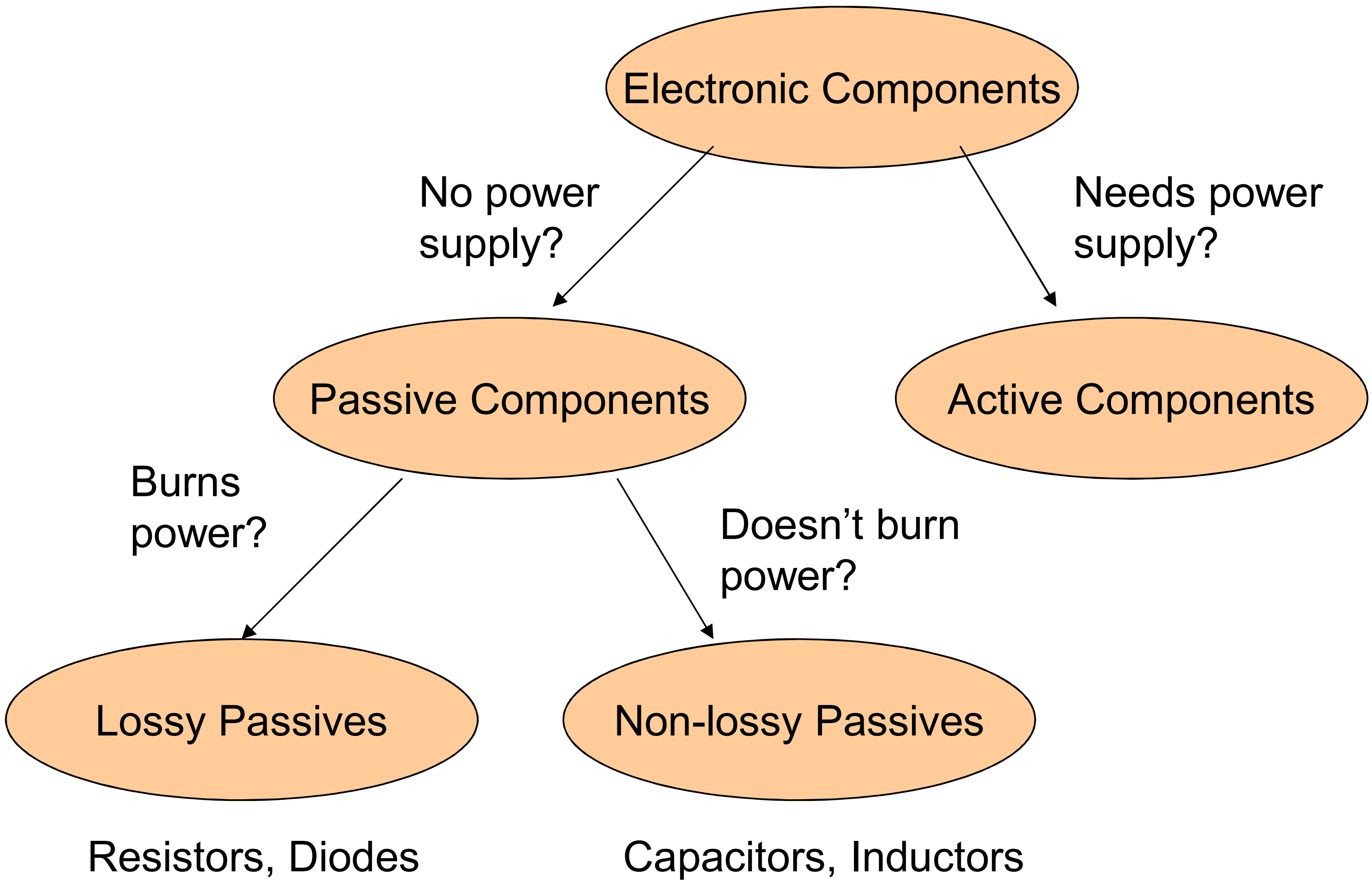
More Circuit Components: Capacitors, Inductors, and Diodes



- Practical circuits contain many components:
- Resistors (discussed last week)
- Capacitors, Inductors, and Diodes (to be discussed this week)
- Op Amps, Timers, etc (to be discussed next week)

Categories of Components:

Passive vs. Active, Lossy vs. Non-Lossy



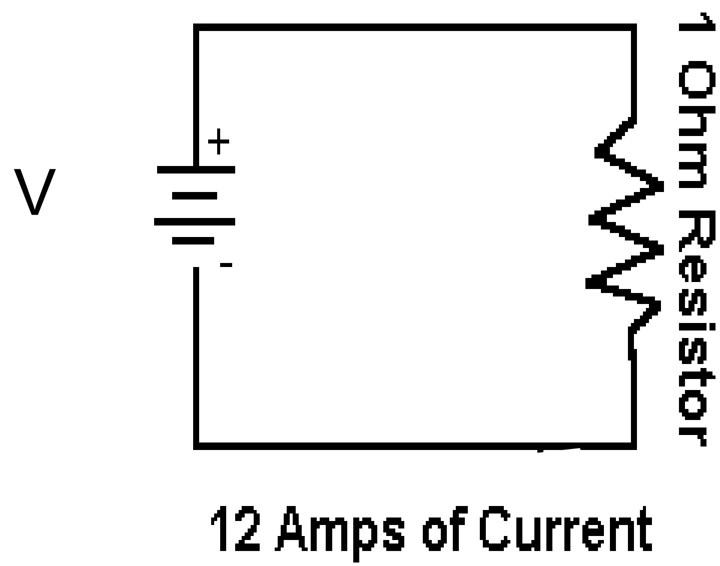
Capacitors and Inductors

- Are not resistors. They:
- Store energy
  - Capacitor in electric field, Inductor in magnetic field
- Are non-lossy (*reactive* vs. resistive)
- Have a time-varying behavior
  - Different from resistors

Lets review Resistors

We had Ohms law:

V = I R



Multiply both sides by current

$IV = I^2R$

I is Charge/second or $\Delta Q/\Delta t$

$(\Delta Q/\Delta t)\ (V) = I^2R$

Or $(\Delta Q\ V)/\ \Delta t = I^2R$

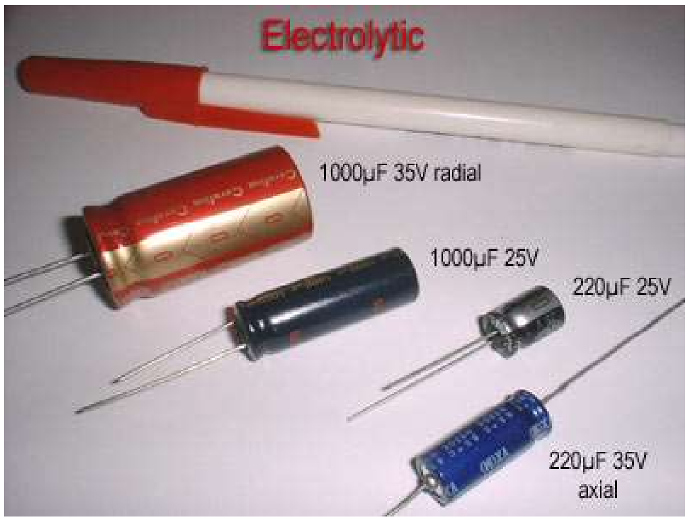
But QV is electrical potential energy (like mgh for gravitational potential energy)

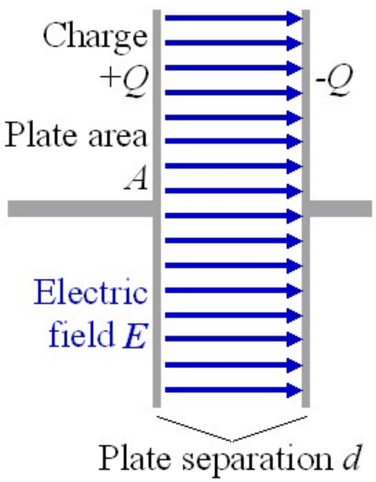
So the energy dissipated per second in a **resistor** is:

Energy/second = I^2R = power

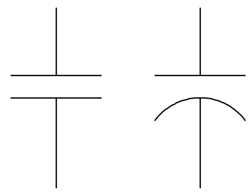
Capacitors

- Capacitors store energy in the form of an electric field
- Constructed from closely spaced conductors
- They act like small rechargeable batteries, able to store and release electrical energy.



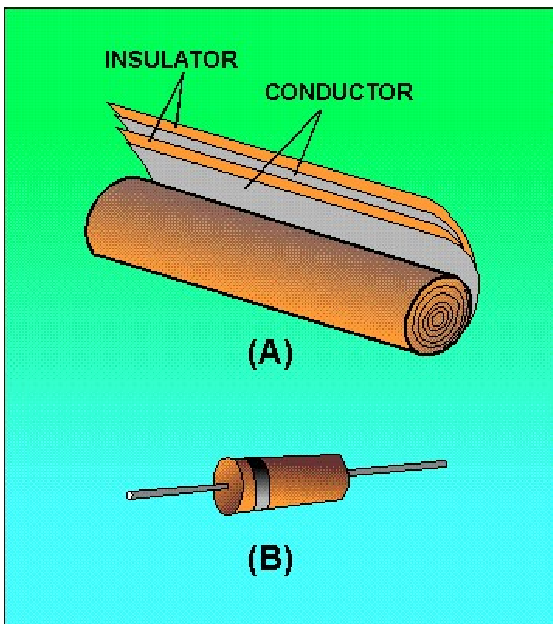


Symbols:

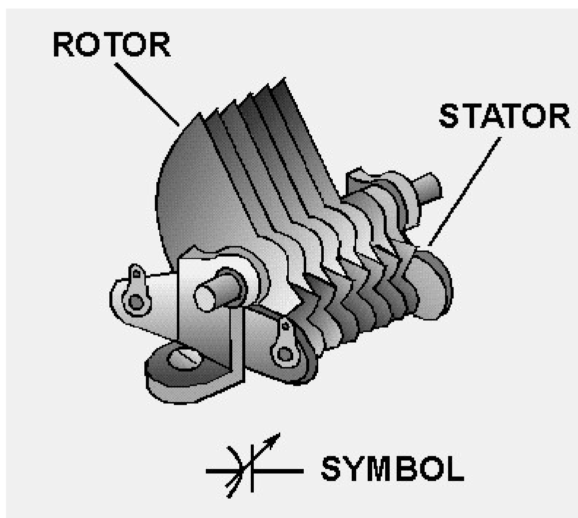


Capacitor Construction

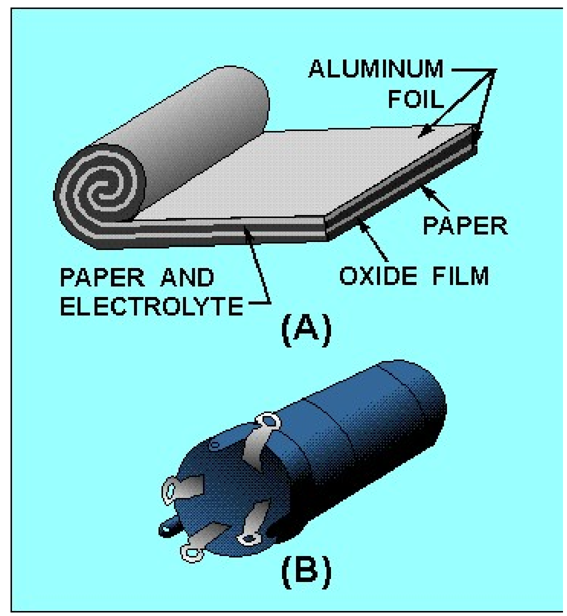
- Different types for different applications
- Choose for capacitance, size, voltage rating, leakage, series resistance...



Metal film capacitor



Variable capacitor



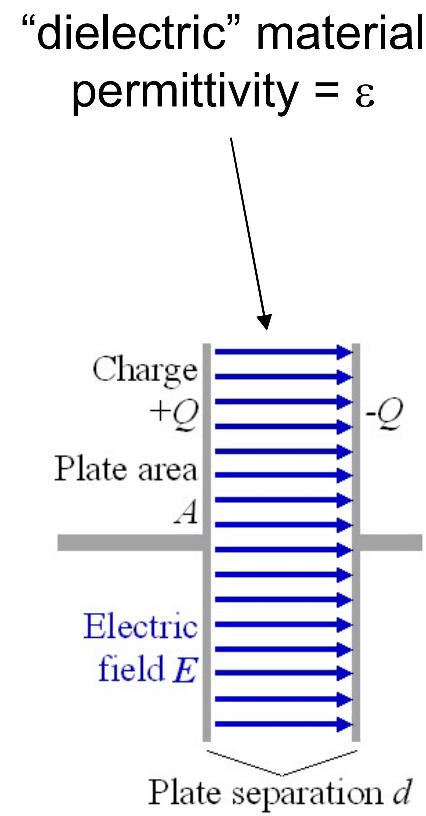
Electrolytic capacitor

Capacitor Equations

- A capacitor holds charge *Q* proportional to the voltage across it:

$$C = \frac{Q}{V}$$

- The *capacitance C* (units of *Farads)* is set by the construction of the capacitor:



$$C \approx \frac{\epsilon A}{d}; A \gg d^2$$

Capacitor behavior

- Current through capacitor proportional to rate of change in voltage across it:

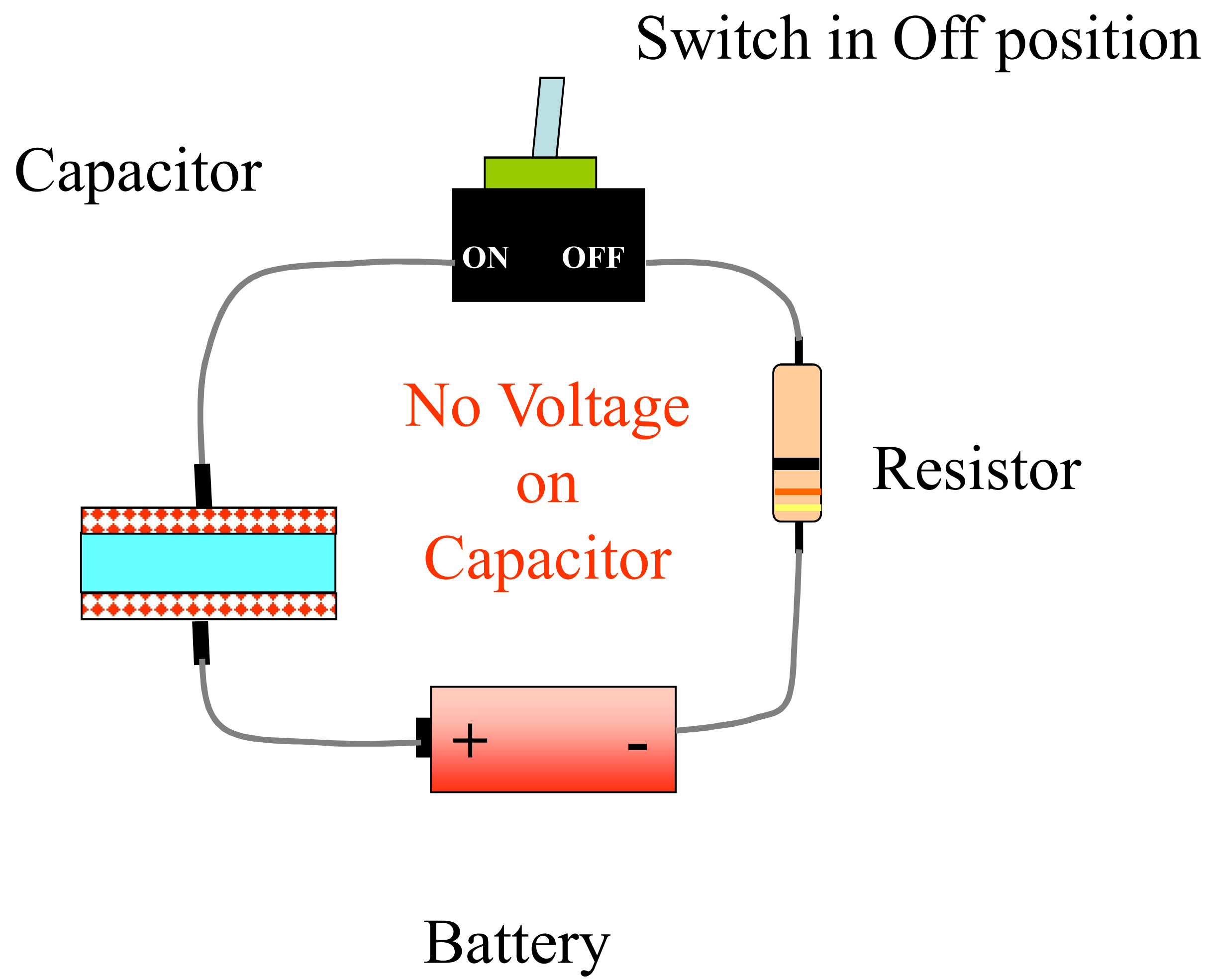
$$i = \frac{\Delta Q}{\Delta t} = C\,\frac{\Delta V}{\Delta t}$$

- Capacitors act to resist changes in voltage
- Capacitor current can change (very) quickly
- Capacitors store energy:

$$E_{\mathrm{stored}} = \frac{1}{2}CV^2 \Leftrightarrow E_{\mathrm{stored}} = \frac{1}{2}\frac{Q^2}{C}$$

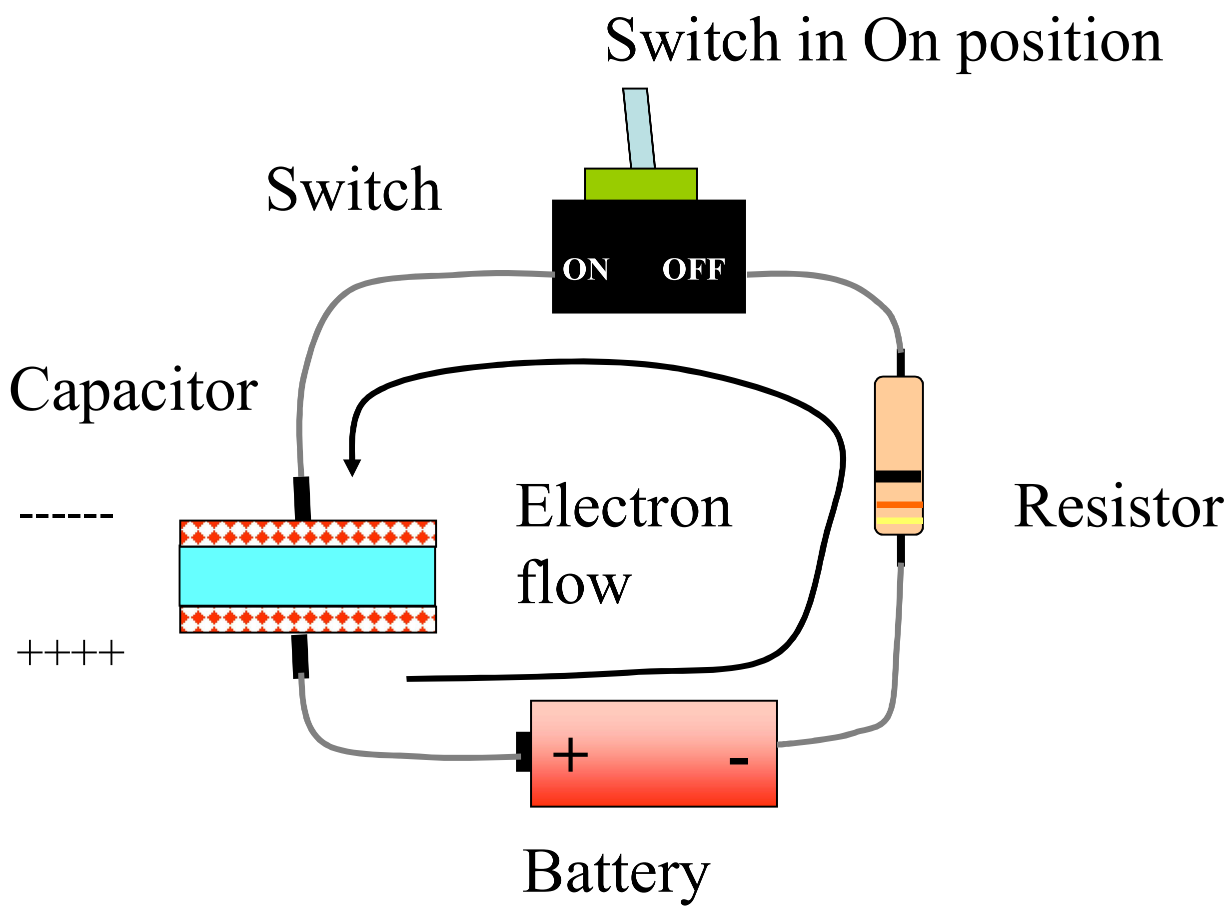
Basic Resistor Capacitor (RC) Circuit

- Initially, capacitor is “uncharged.”

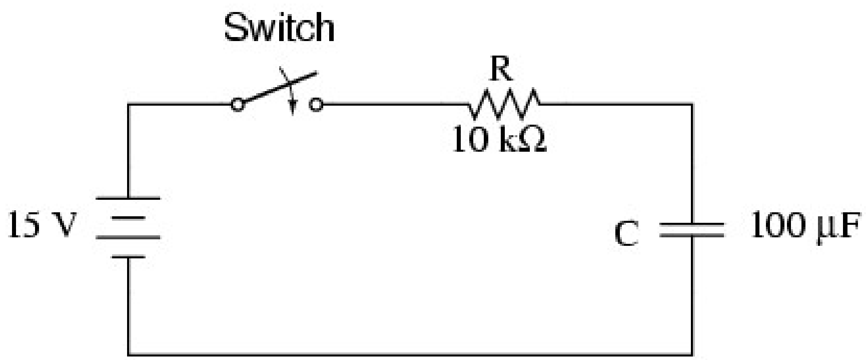


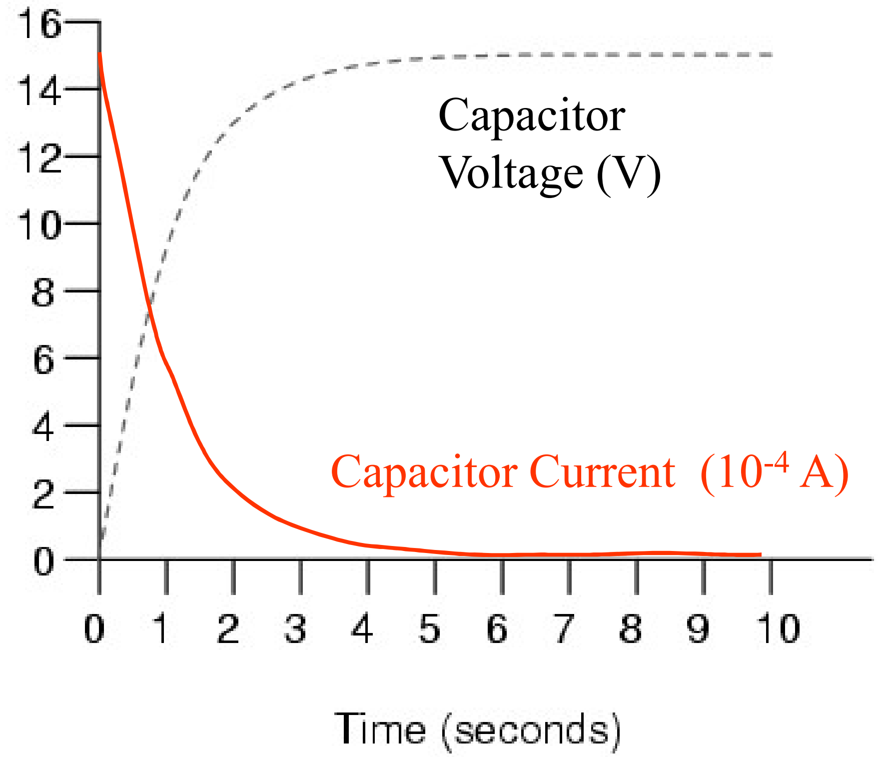
Basic Resistor Capacitor (RC) Circuit

- When current flows, capacitor becomes charged to the voltage of the battery



RC circuit: Time response

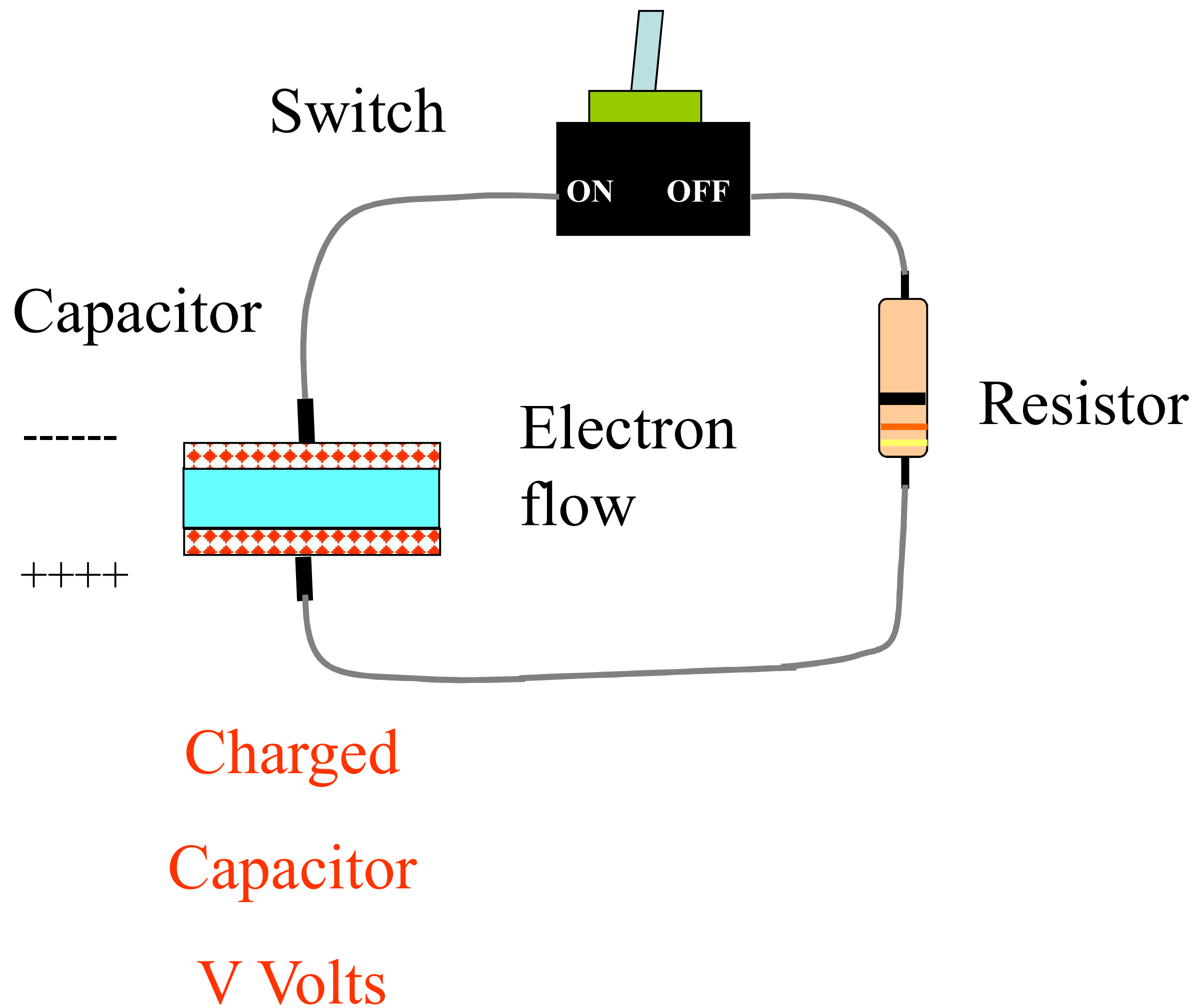




- Inherent time scale is: τ = RC (RC time constant)
- Capacitors act to resist changes in voltage
- Capacitor *current* can change quickly

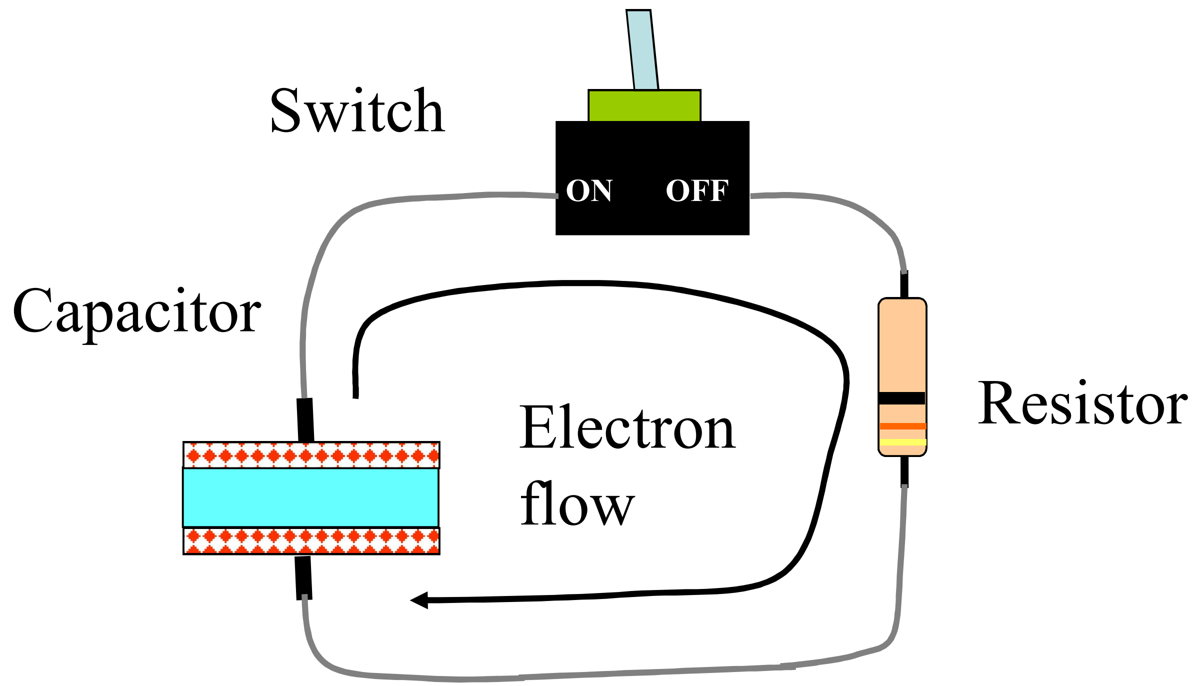
A capacitor stores energy

- And can release it very quickly



A capacitor stores energy

- And can release it very quickly



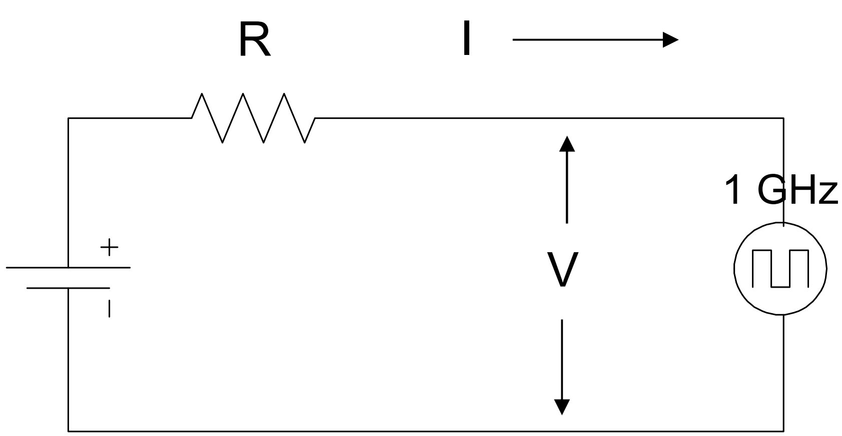
Discharged

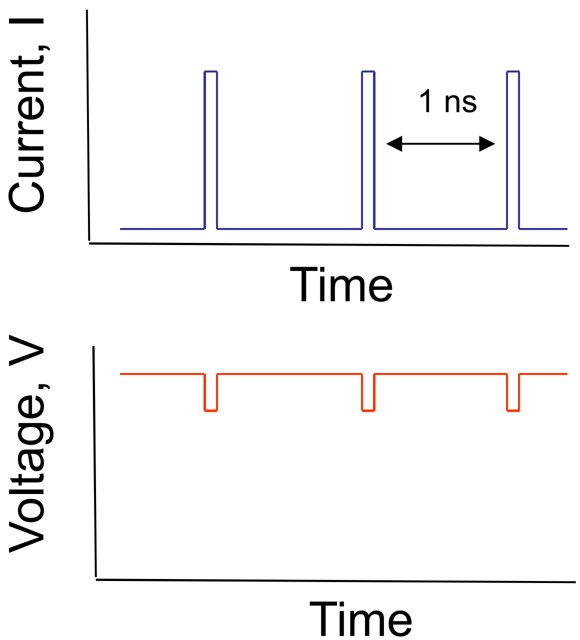
Capacitor

0 V

“Bypass” capacitors

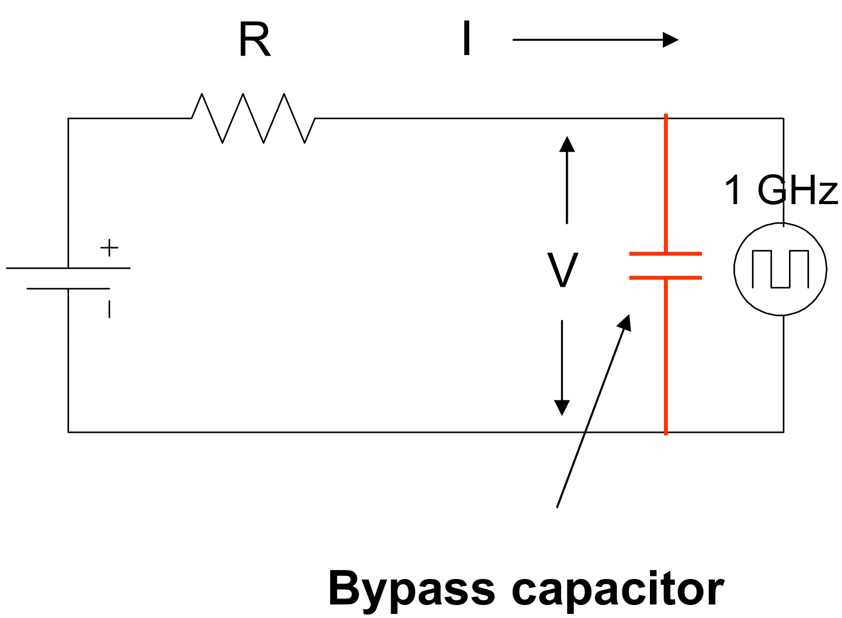
- Digital electronics contain many transistor switches which require bursts of current when they switch
- Can cause troublesome power supply noise

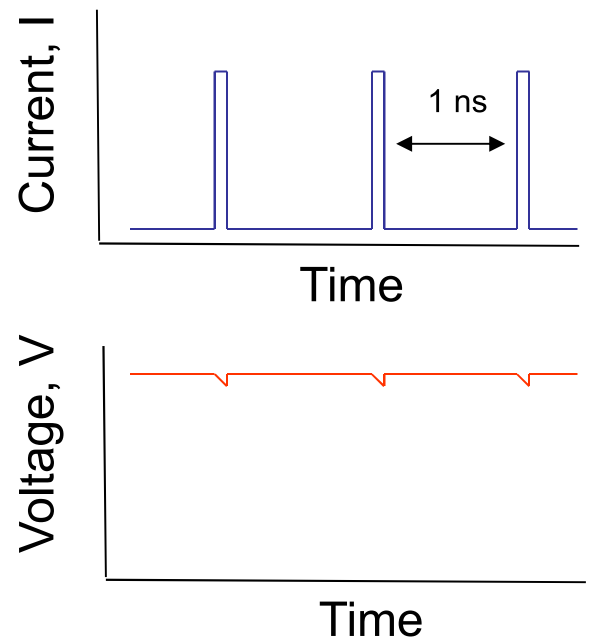




"Bypass" capacitors

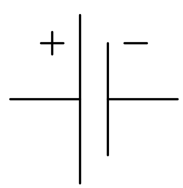
- A small capacitor, close to and in parallel with a digital device smoothes out the voltage by providing a low resistance "backup" voltage source



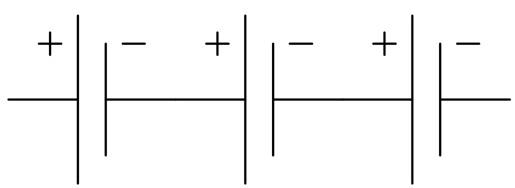


Voltage multiplication with capacitors

- Resistors can only decrease circuit voltages
- Capacitors can be used like batteries in series to increase voltages...



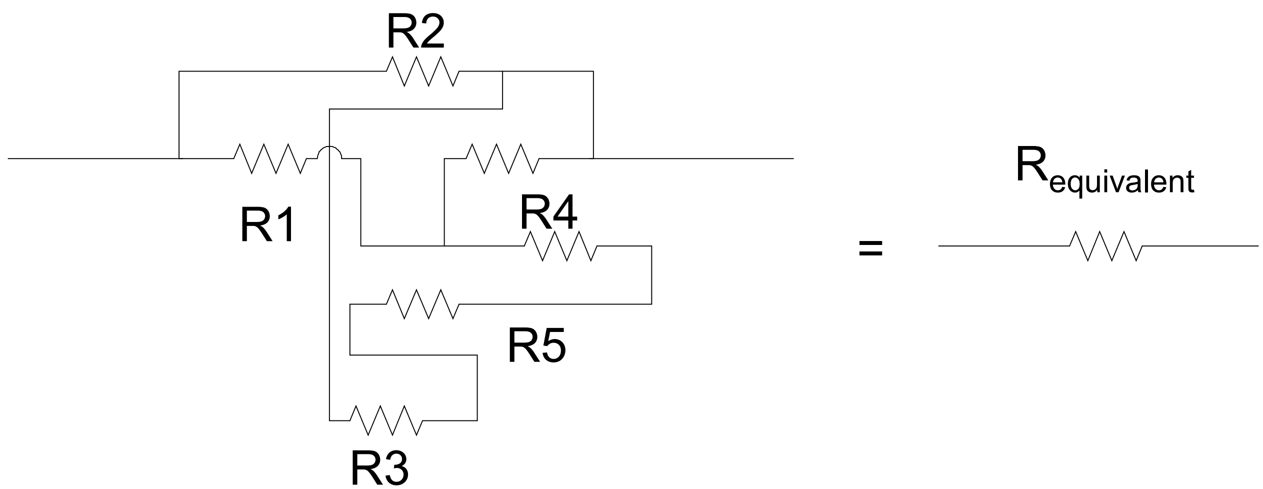
9V



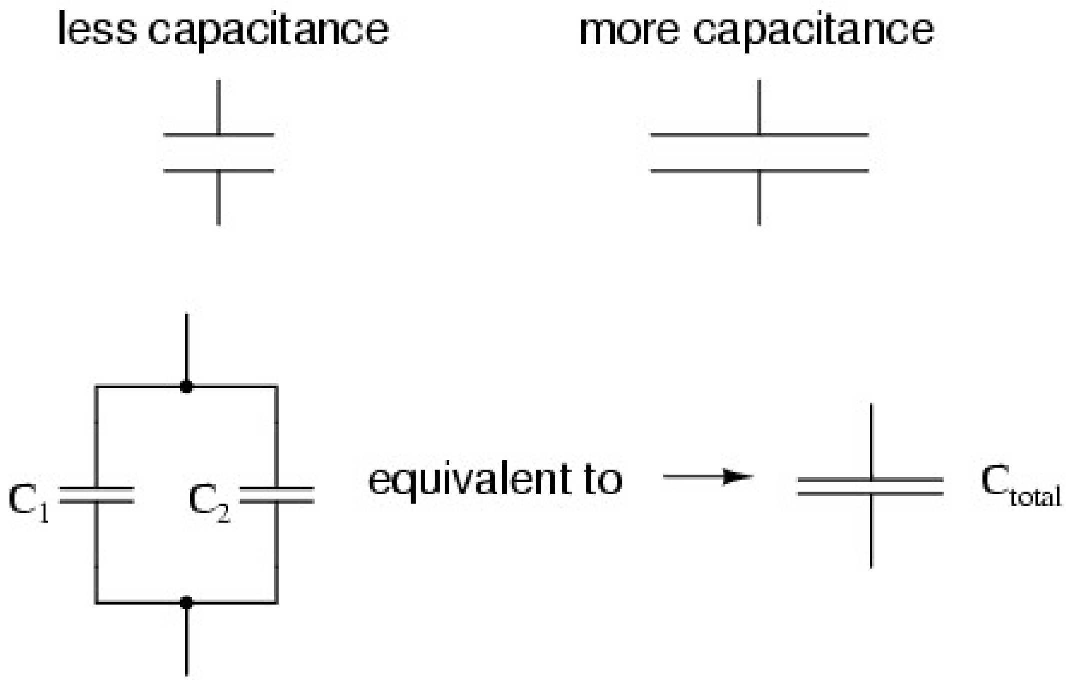
9V x 3 = 27V

Series and Parallel Combinations

- Any circuit constructed using just resistors also acts like a resistor. Same for capacitors and inductors.
- The circuit has an equivalent resistance (or capacitance, or inductance) which depends upon the details of the network



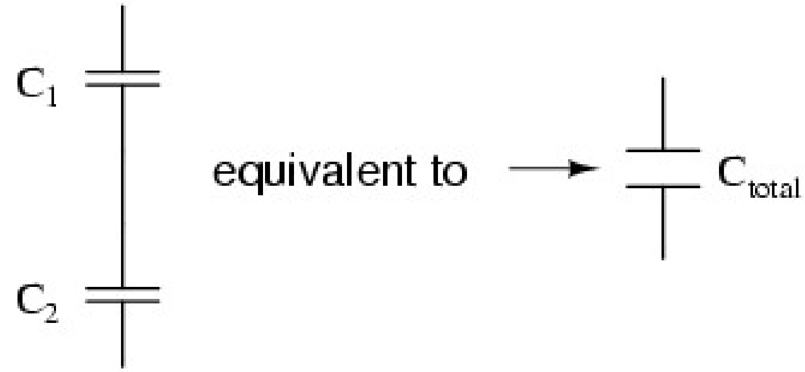
Parallel Capacitors



Parallel Capacitances

$$C_{total} = C_1 + C_2 + \ldots C_n$$

Series Capacitors



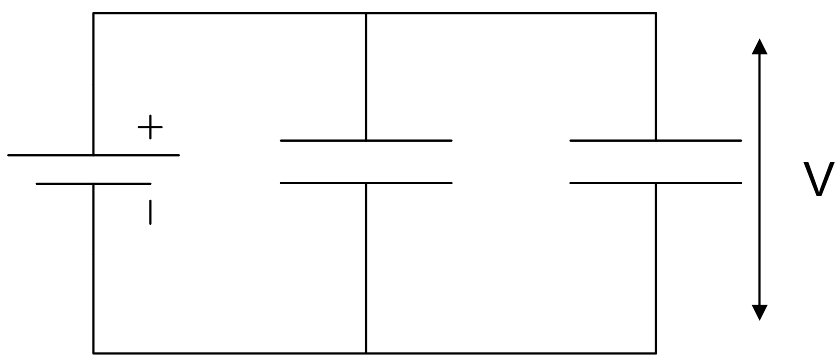
less capacitance more capacitance

Series Capacitances

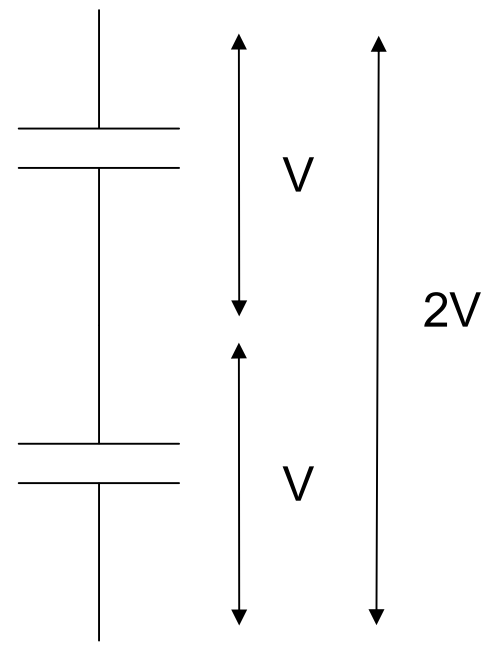
$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots \frac{1}{C_n}}$$

Switched-capacitor circuits

- Switch circuits can "rearrange" capacitors to achieve many useful effects...
- Example: voltage doubler



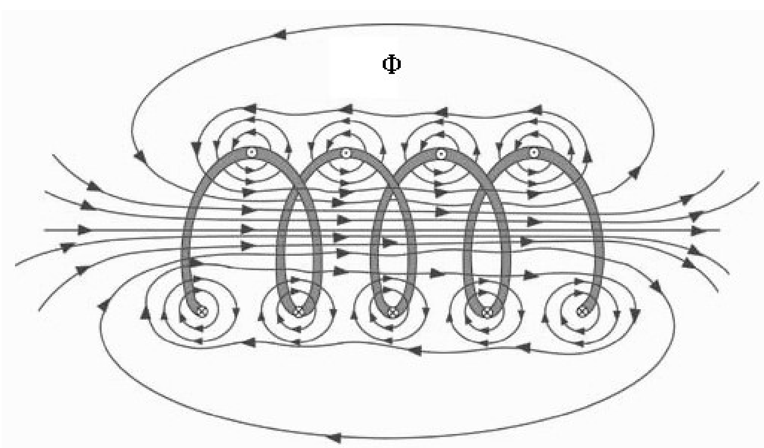
Step 1: Charge two capacitors in parallel

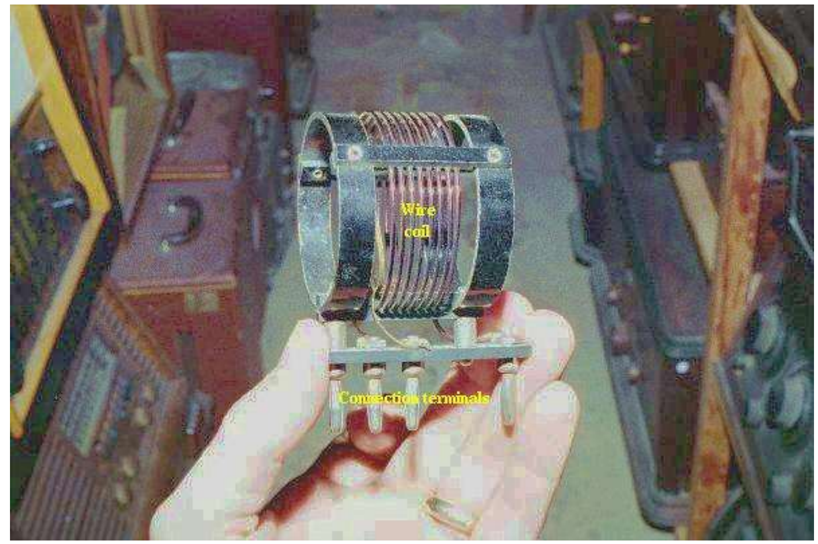


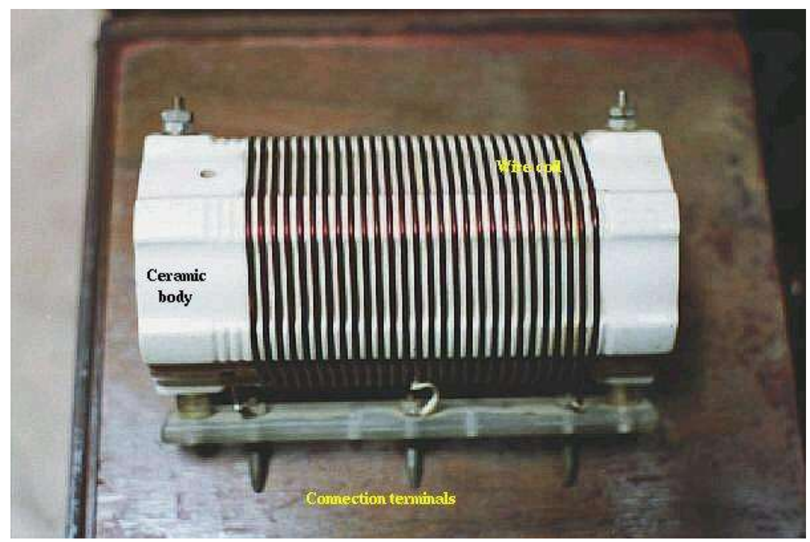
Step 2: Switch the two capacitors to be in series

Inductors

- Inductors store energy in the form of a *magnetic field*
- Made from coils of wire

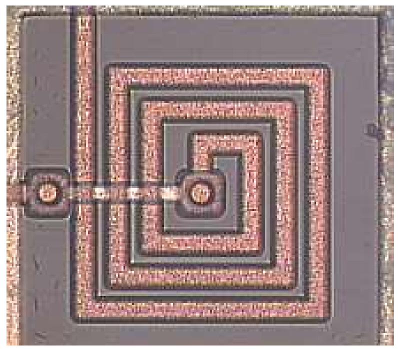


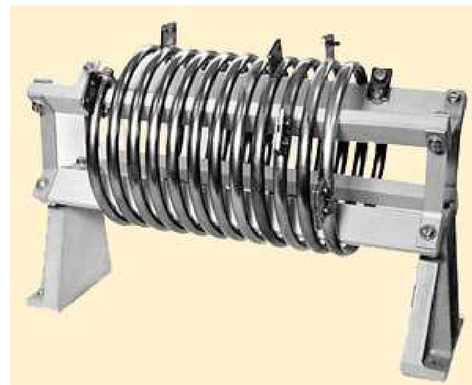


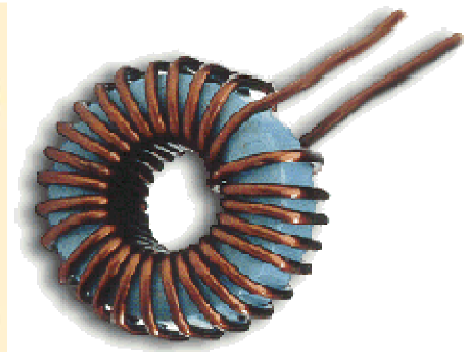


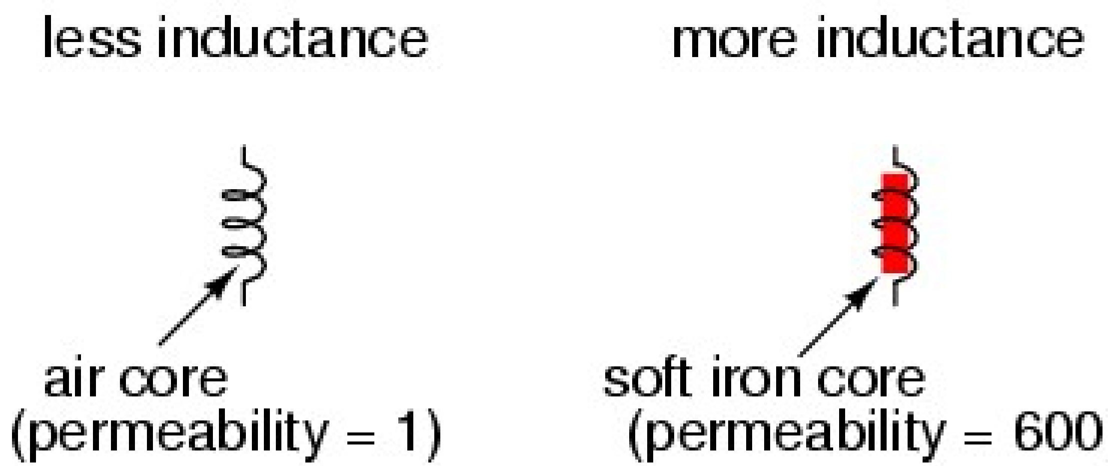
Inductor construction

- Can have various geometries, big and small
- Coils can be wrapped around *high-permeability* materials to increase inductance
- It is a lot harder to make a good inductor than a good capacitor or resistor
  - Lossy
  - Big
  - “Parasitics”
- Inductors are mostly used for only a few applications
  - Radio circuits
  - Power conversion



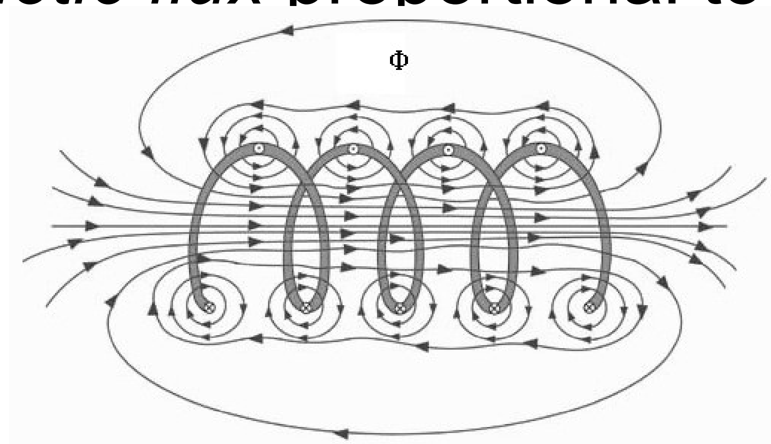






Inductor Equations

- An inductor stores *magnetic flux* proportional to the current through it:



$$L = \frac{\Phi}{i}$$

- The inductance L (units of *Henries)* is set by the construction of the inductor:

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

μ_0 = permeability of free space = 4π × 10-7 H/m
μ_r = relative permeability of core material
N = number of turns
A = area of cross-section of the coil in square meters (m^2)
l = length of coil in meters (m)

Inductor behavior

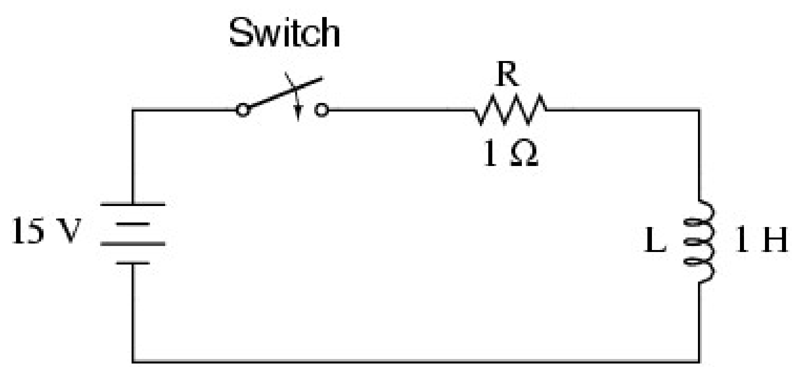
- Voltage across inductor is proportional to the rate of change of current through it:

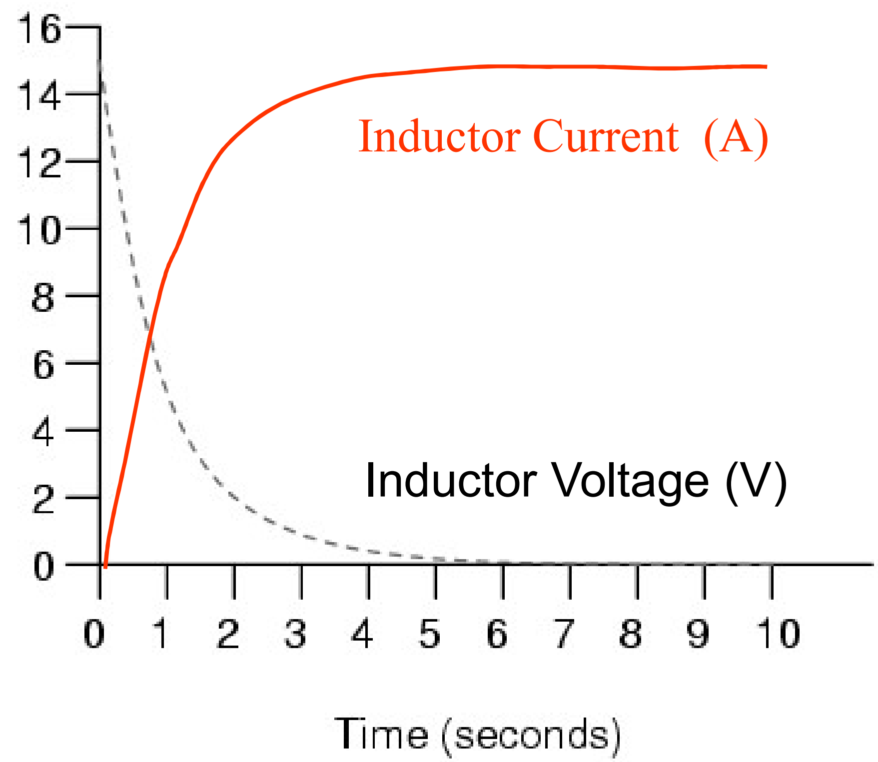
$$V = L\frac{\Delta I}{\Delta t}$$

- Inductors act to resist changes in current
- Inductor voltage can change quickly
- Inductors store energy:

$$E_{\text{stored}} = \frac{1}{2}LI^2$$

Inductor/Resistor (RL) Circuit





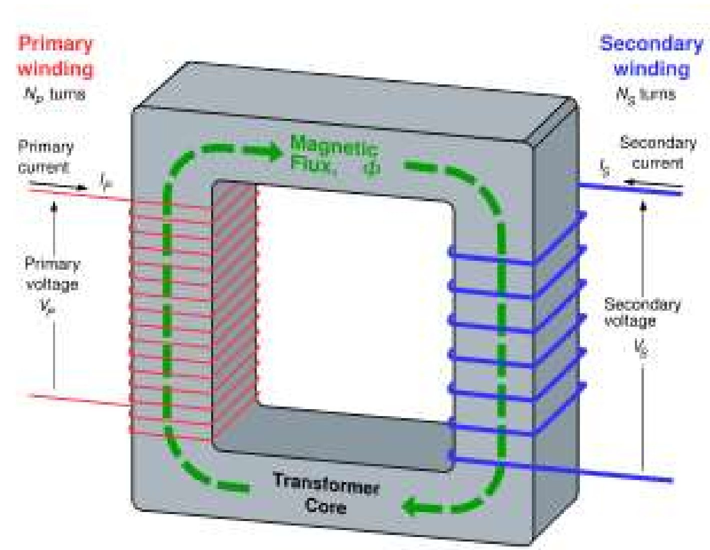
- Inductors act to resist changes in *current*
- Inductor *voltage* can change quickly
- Inherent time scale is: $\tau = L/R$

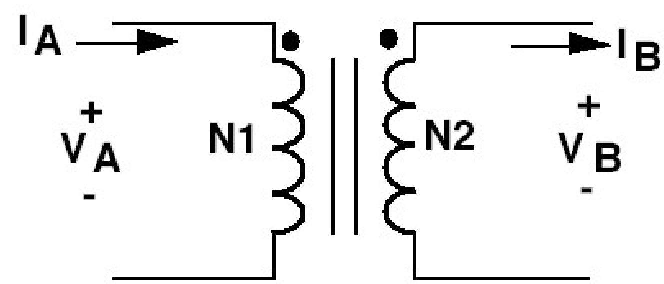
Inductor Applications: Filtering

- Because inductors tend to reject the flow of rapidly changing currents, they can be used to filter out high-frequency signals
- Ex: Lumps on computer cables are *ferrite beads* which add inductance to the cable.

Transformer

- An inductor's magnetic field can pass through more than one coil
- This is called a t*ransformer.*
- For AC (time-varying) signals, it transforms the voltage observed between its coils according to its *turns ratio.*
- A transformer *isolates* the two sides of the transformer – no physical connection other than through the magnetic field. This is often desirable for safety reasons.

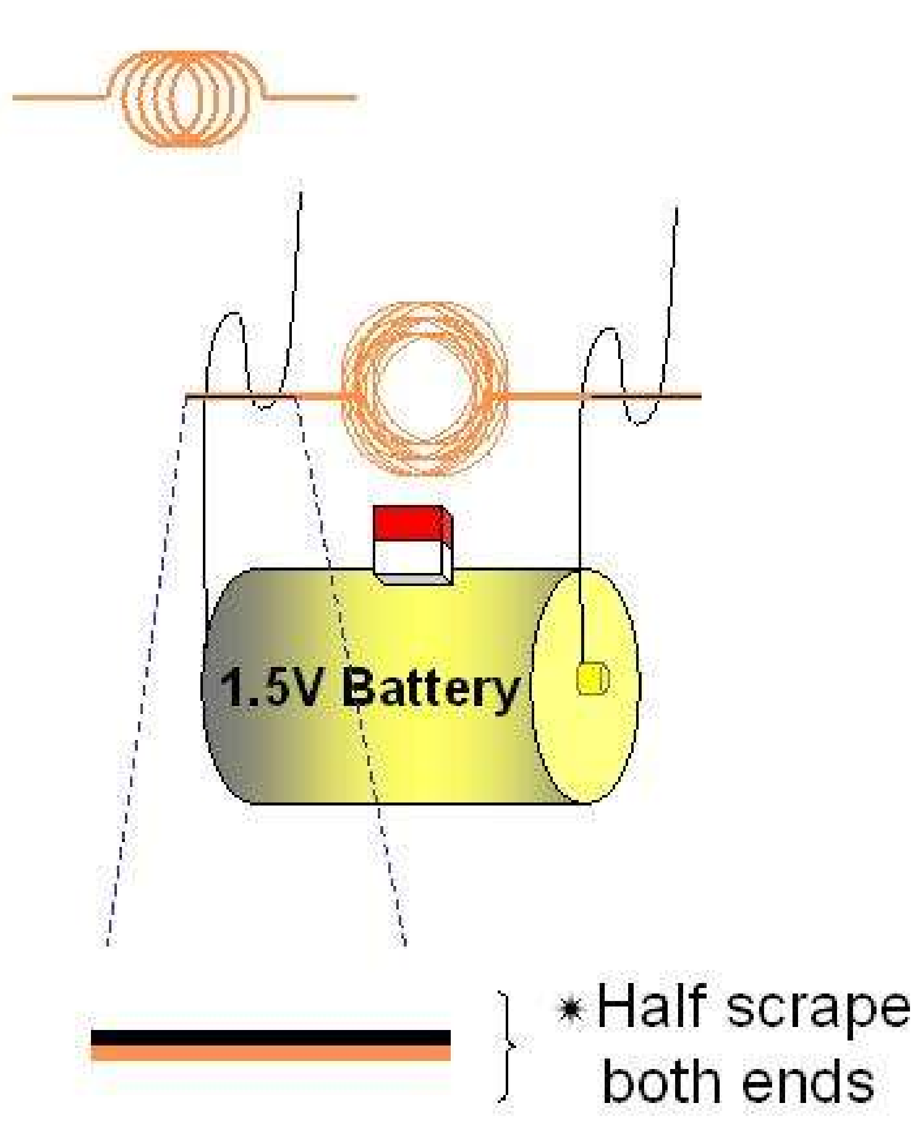




$$V_B = V_A \frac{N2}{N1} \qquad I_B = I_A \frac{N1}{N2}$$

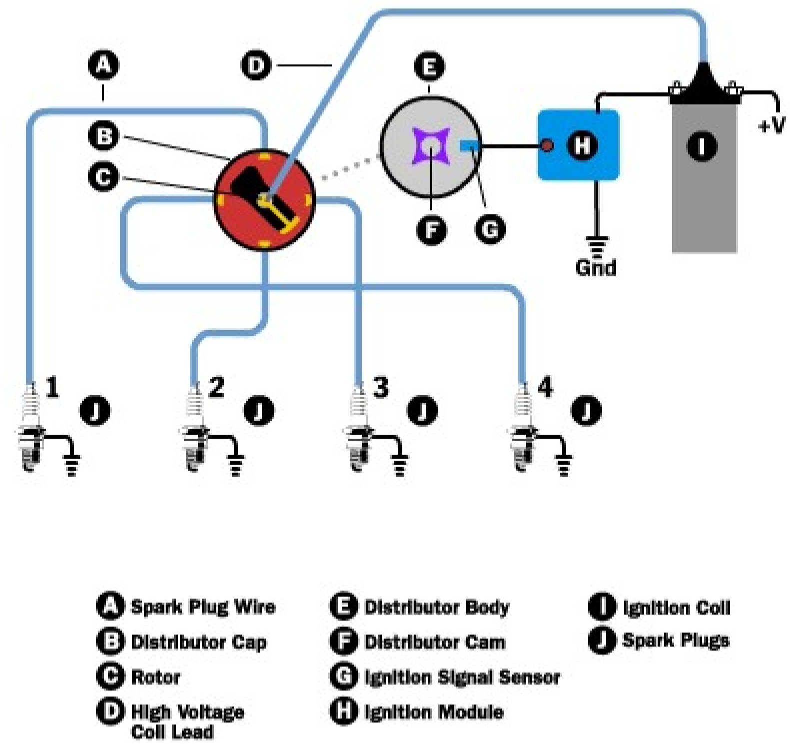
Build Your Own Electric Motor

- Take a length of coated .5mm diameter wire and wrap into a coil
- Using a razor or scissor: scrape the coating off ½ of the ends ✱
- Tape bent paperclips to ends of 1.5V battery using electrical tape
- Place small magnet on top of battery
- Give coil a little spin and watch it go!

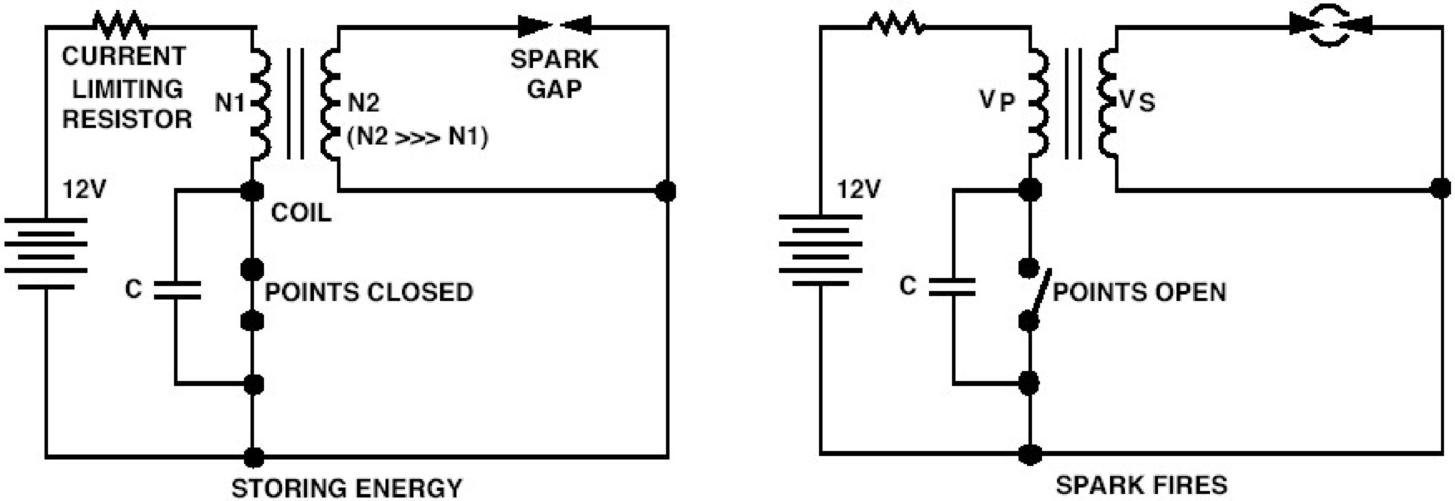


Engine Ignition Circuit

- A transformer is used to generate 40,000V to fire the engine spark plugs



Ignition Schematic

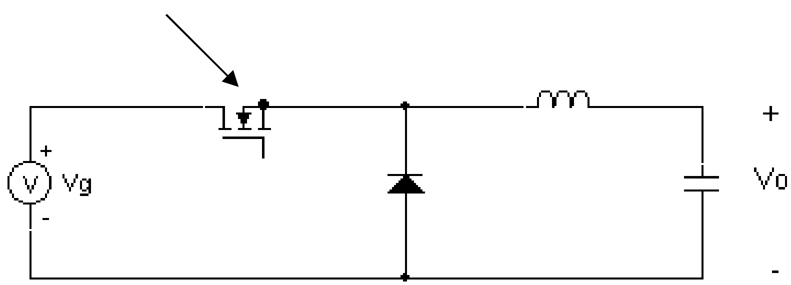


- “Coil" has high secondary-to-primary turns ratio.
- When the points first close, current starts to flow in the primary winding and eventually reaches the final value set by the 12V battery.
- When the points open, the current in the primary winding collapses very quickly, causing a large voltage to appear across this winding. This voltage on the primary is magnetically coupled to (and stepped up by) the secondary winding, generating a voltage of 30 kV - 40 kV on the secondary side.
- In the automobile ignition, a capacitor is placed across the points to minimize damage due to arcing when the points "break" the current flowing in the low-voltage coil winding (in car manuals, this capacitor is referred to as a "condenser")

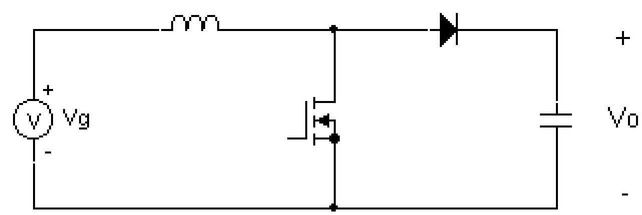
Inductor Applications: Switching power supplies

- Inductors make possible many types of efficient DC-DC converters which convert an input voltage to a different voltage (either higher or lower)
- Can be >90% efficient

Switch: on “D”% of the time
(D = duty cycle)



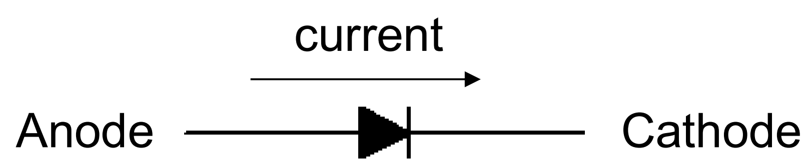
“Buck” configuration
Vo/Vg = D



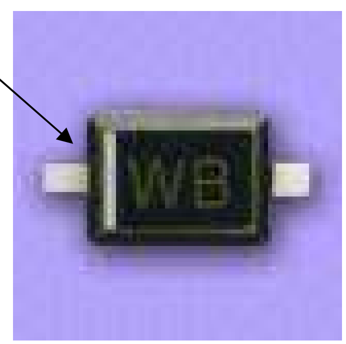
“Boost” configuration
Vo/Vg = 1/(1-D)

The Diode: A one-way street

- Diodes are electronic components that conduct current in one direction only, from *anode* to *cathode*

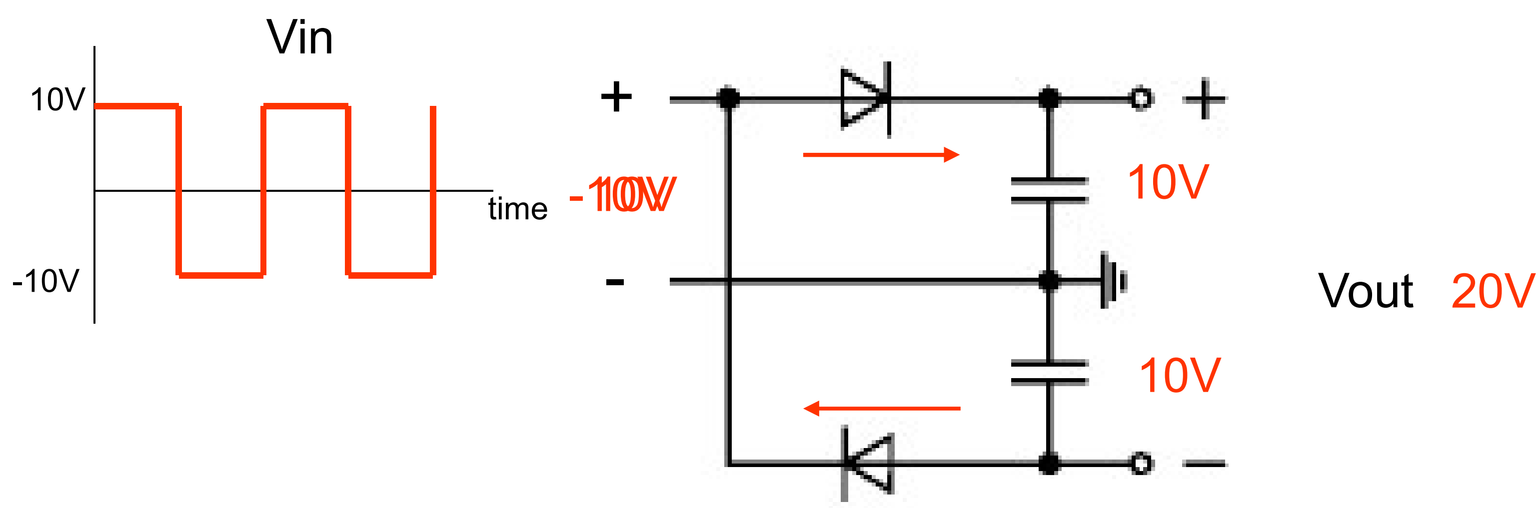


Cathode



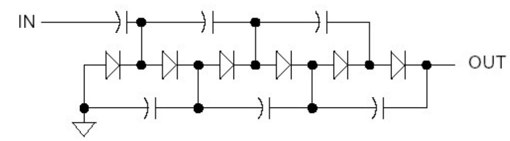
The diode as a switch

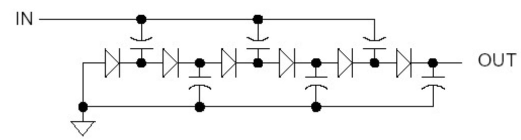
- Diodes can be used as passive switches – to conduct positive voltages along one path and negative voltages along another
- Ex: voltage doubler

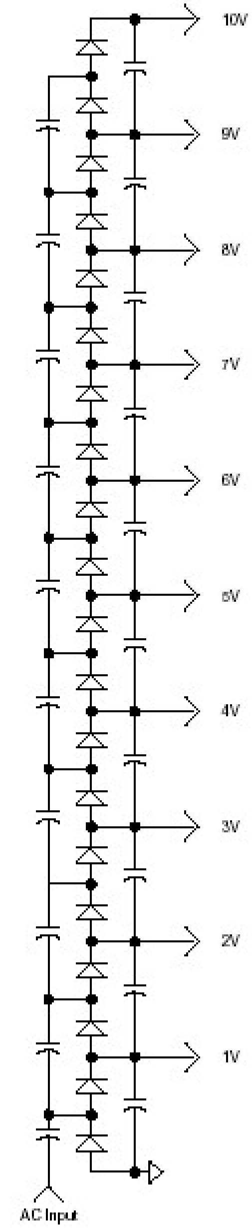


Voltage multipliers

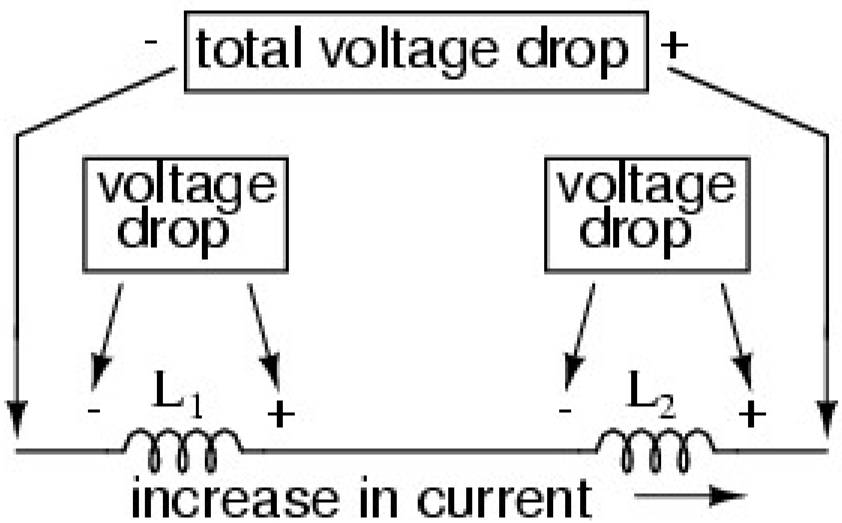
- The voltage doubler principle can be generalized to make voltage *multipliers* capable of generating very high voltages
- One popular type was developed by Cockcroft and Walton to produce 800 kV needed for their particle accelerator (1932)
- Nobel Prize: First demonstration of a nuclear reaction caused by accelerated particles







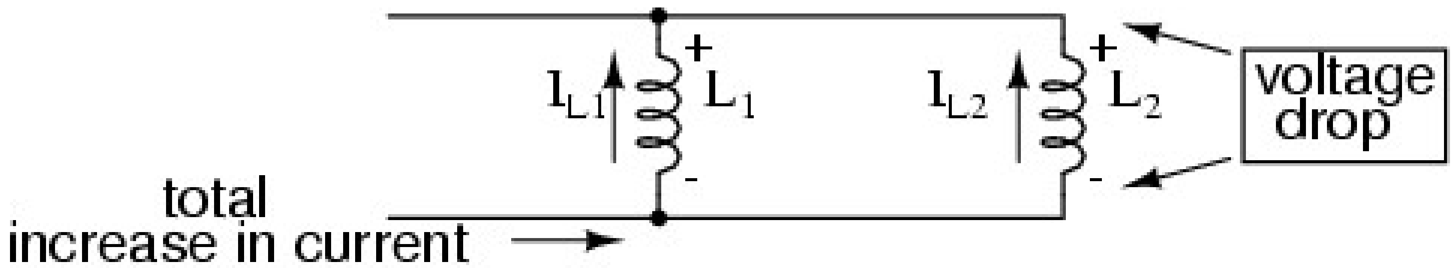
Series Inductors



Series Inductances

$$L_{total} = L_1 + L_2 + \ldots L_n$$

Parallel Inductors



Parallel Inductances

$$L_{total} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n}}$$